Effects of a Fluctuating Interface between a Superfluid and a Polarized Fermi Gas

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Motivated by recent experiments in trapped Fermi gas with spin population imbalance, we discuss the effects of the quantum and thermal fluctuations of the interface between a fully paired superfluid core and a fully polarized Fermi gas. We demonstrate that even if there is no true partially polarized thermodynamic phase in bulk, the interface fluctuation can give rise to a partially polarized transition regime in trap. Our theory yields a definite prediction for the functional forms of the spatial profile of spin polarization and pairing gap, and we show that the spin-resolved density profiles measured by both the MIT and Rice groups obey this function form. We also show that sufficient large fluctuation will lead to a visibly unequal density even at the center of the cloud. We hope this picture can shed lights on the controversial discrepancies in recent experiments.

Phase separation is an ubiquitous natural phenomenon. Its root is first order phase transition. For example, as a function of temperature (T) and magnetic field (H), an easy-axis ferromagnet phase separates into two oppositely magnetized phases when (T, H) are *fined tuned* to the boundary of the first-order transition ($H = 0, T = T_c$). On the other hand, as long as $T < T_c$ phase separation occurs without needing fine tuning when the magnetization, rather than magnetic field, is fixed. In the presence of phase separation the dynamics of the interface determines the low energy/temperature physical properties of the system. Recently experiments of ultracold Fermi gases near Feshbach resonance once again reveal phase separation phenomenon[1, 2, 3]. In these experiments, an external-imposed spin population imbalance frustrates the strong pairing interaction due to Feshbach resonance. As a result, the system phase separates into a core of superfluid with equal spin population and a shell of excess unpaired fermions.

Although similar phenomena were observed, there are important difference between the findings of two major experimental groups at MIT and Rice University. Let $N_{\uparrow,\downarrow}$ represent the number of spin up/down atoms, and $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow)$ be the population imbalance. Here are the highlight of the differences. The MIT group finds (i) for small P, the spin-resolved density profile exhibits three regions: a superfluid core with $n_\uparrow = n_\downarrow$, a partially polarized (PP) shell with $n_\uparrow > n_\downarrow > 0$ and a fully polarized region with $n_\downarrow = 0$. The width of the PP shell is as wide as the other two regions (see Fig.5(a) of Ref.[2]). (ii) The interfaces between different regions always follow the equal potential contour as predicted by theories using local density approximation. (iii) As P exceeds a critical value $P_c \sim 70\%$, the equal density core disappears and also the condensation fraction vanishes, which is referred to as the Chandrasckhar-Clogston (CC) limit of superfluidity.[2] In contrast, the Rice group finds (i) the width of the PP region is much narrower than the width of other two regions (see Fig.3(b) of Ref.([3])). (ii) At large P the interface does not follow the equal potential contour. (iii) The equal density core exists up to the highest P studied ($\sim 95\%$), and there is no sign of CC limit.[3] One can expect the explanation of these discrepancies to shed light on a number of fundamental issues of this problem. For examples, what is the nature of the PP region, does it reveal a true quantum phase of the system, and what controls its width; does the CC limit for superfluidity exists, and if so, what is the mechanism.

These experiments have stimulated much theoretical discussions in the literature. Most of the existing theories assume the system to follow the confining potential *adiabatically*, so that locally it corresponds to a bulk phase with the chemical potential $\mu_{\uparrow,\downarrow} = \mu_{0\uparrow,\downarrow} - V(\mathbf{r})$, where $\mu_{0\uparrow,\downarrow}$ is a spin-dependent constant and $V(\mathbf{r})$ is the trapping potential. The issue at hand is whether the PP state is a stable bulk phase near Feshbach resonance. By comparing mean-field energies, many authors predicted a narrow window of PP phase in the bulk phase diagram[6]. However due to the approximate nature of the treatment, and the fact that even in mean-field theory the energy difference between different phases is tiny, it is hard to

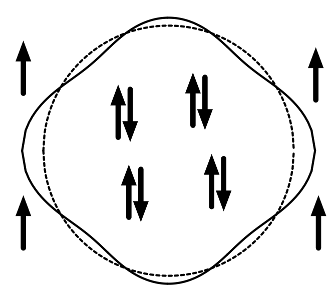

FIG. 1: Schematic of the interface flucuation. The dashing line is the static location of the interface which separates a fully paired superfluid phase and a fully polarized fermi gas. The solid line sketches an example of surface mode which distorts the interface.

know whether such a PP bulk phase really exists or not.

In the following we make a bold assumption: the PP region found in experiments is *not* a true phase, rather it is due to the fluctuations of the interface between the equal-population superfluid and the fully polarized phase. Our assumption implies that, for interaction strength relevant to the experimental systems, there is a direct zero-temperature first order transition between the superfluid and the fully polarized phases as a function of the Zeeman field. Our proposal is motivated by two recent developments. (i) Recently the MIT group found the presence of a full pairing gap in the PP state at $P > P_c$ where the superfluidity is destroyed[4]. This contradicts theories which attribute the CC limit to depairing[5]. (ii) The importance of interface physics is highlighted by recent theoretical works which attribute the deformation of the interface from the equal potential surface to the existence of finite surface tension[7, 8]. Of course, interface physics involves not only statics but also dynamics .

In the rest of the paper we analyze the effect of *harmonic* interface fluctuations between the superfluid and the fully polarized phase. In Fig.1 we illustrate a snap shot of an allowed interface deformation where the enclosed volume is kept fixed. Surface fluctuations have been studied in early days of atomic BEC and fermion superfluid with equal population[9]. However since the system is consisted of only one single phase, such fluctuation is less important as it only affects the low density region at the edge of the cloud. In comparison the effects of interface fluctuation are far more pronounced here which lead to a very precise and universal profile of spin polarization and superfluid order parameter across the two phases.

Here are how we describe the interface. i) Motivated by the experimental facts[2, 3], we assume the equilibrium interface $\mathcal{S}$ is a closed two-dimensional surface with cylindrical symmetry:

$$\mathcal{S} = \{(x, y, z) : x^2 + y^2 + \epsilon^2(z)z^2 = a^2\}. \qquad (1)$$

Here $\epsilon(z)$ is an even function, its value is unity for $z = 0$, and decreases monotonically to $a/z_0 < 1$ as z approaches $\pm z_0$, the north and south poles of $\mathcal{S}$. In the following we shall use z and the azimuthal angle ϕ as the coordinates for $\mathcal{S}$, so that $x = \sqrt{a^2 - \epsilon^2(z)z^2}\cos\phi$, $y = \sqrt{a^2 - \epsilon^2(z)z^2}\sin\phi$, $z = z$. It should be apparent that this is a locally orthogonal coordination, i.e., $ds^2 = g_\phi(z)d\phi^2 + g_z(z)dz^2$. (ii) We describe the fluctuation of the interface from its equilibrium shape by a *harmonic* quantum elastic model. Let $u(z, \phi, t)$ be the time-dependent *normal displacement* of the interface from its equilibrium position, the Lagrangian of our model reads

$$\mathcal{L} = \int_{\mathcal{S}} d\phi dz \sqrt{g_\phi g_z}\Big[\frac{m}{2}\dot{u}^2 - \frac{V(z)}{2}u^2 - \sum_{\mu=\phi,z}\frac{(\partial_\mu u)^2}{2g_\mu}\Big], \qquad (2)$$

where m is an introduced phenomenological parameter. The normal modes of the interface fluctuation obey the following differential equation

$$\frac{1}{\sqrt{g_\phi g_z}}\partial_z\left(\sqrt{\frac{g_\phi}{g_z}}\,\partial_z u_\alpha\right) + \frac{\partial_\phi^2 u_\alpha}{g_\phi} - V(z)u_\alpha = -m\omega_\alpha^2 u_\alpha, \qquad (3)$$

where ω_α is the normal mode frequency. In later analysis we shall expand $u(z, \phi, t)$ in terms of the normal modes :

$$u(z, \phi, t) = \sum_\alpha{}' A_\alpha(t)u_\alpha(z, \phi). \qquad (4)$$

Note that all quantities in the above equation are real. In Eq. (4) the sum excludes the "breathing" mode, the mode whose associated distortion does not conserve the enclosed volume. Interestingly, despite the presence of $g_\phi(z)$ and $g_z(z)$ in Eq. (2) and Eq. (3), the functional form of the interface profile can be determined exactly along special directions such as the z-axis[3] and any radial direction in the $z = 0$ plane[2].

For example, as the main result of this paper, we obtain the following function form for the spin polarization $M = n_\uparrow - n_\downarrow$ and the pairing gap profiles along a high symmetry radial direction ($\hat{t}$):

$$\begin{aligned} M(r,\hat{t}) &= \frac{1}{2}\left[1 + \mathrm{Erf}\left(\frac{r - r_0(\hat{t})}{\xi(\hat{t})}\right)\right] n_\mathrm{p}(r,\hat{t}) \\ \Delta(r,\hat{t}) &= \frac{1}{2}\left[1 - \mathrm{Erf}\left(\frac{r - r_0(\hat{t})}{\xi(\hat{t})}\right)\right] \Delta_\mathrm{s}(r,\hat{t}). \end{aligned} \qquad (5)$$

Here $\mathrm{Erf}(x)$ is the error function, $n_p(r,\hat{t})$ is the density profile of a fully polarized Fermi gas, and $\Delta_\mathrm{s}(r,\hat{t})$ is the pairing gap profile of the fully-paired superfluid, along the same direction in the same trap. In particular we use[11]

$$\begin{aligned} n_\mathrm{p}(r,\hat{t}) &= n_0[\ 1 - (r/R(\hat{t}))^2\ ]^{3/2} \\ \Delta_\mathrm{s}(r,\hat{t}) &= \Delta_0[\ 1 - (r/R(\hat{t}))^2\ ]. \end{aligned} \qquad (6)$$

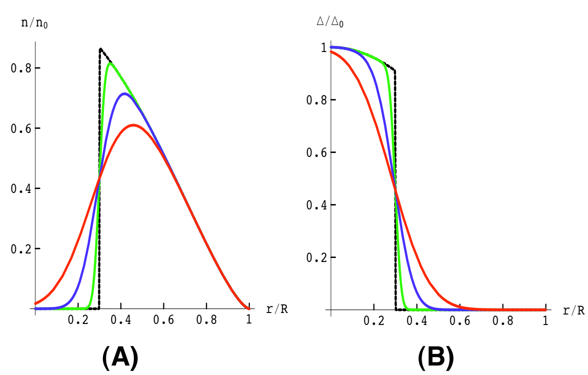

FIG. 2: Plots of Eq. (5) and Eq. (6) for different choices of ξ/R. We have use $r_0/R = 0.3$ for all curves. $\xi/R = 0.03$ for green line, $\xi/R = 0.1$ for blue line and $\xi/R = 0.2$ for red line. The black dashing lines are the profiles without interface fluctuation, which exhibit discontinuity.

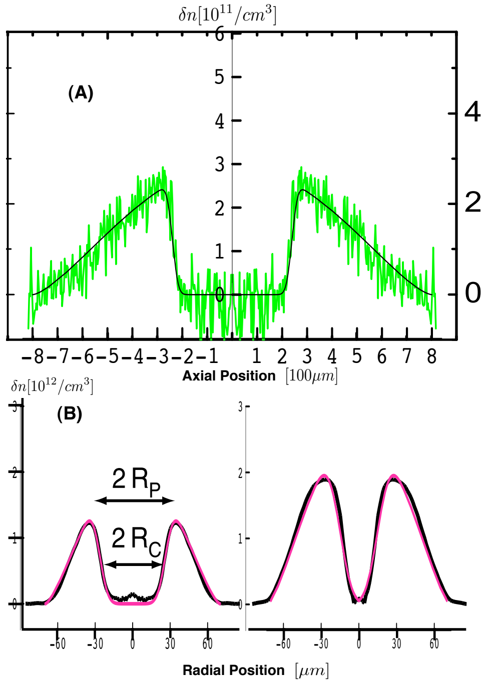

FIG. 3: Fit the measured spin polarization profile to Eq. (5) and Eq. (6). The green line in (A) and the black line in (B) are the data of the Rice[3] and MIT[2] group, respectively. The black line in (A) and the purple line in (B) are our best fits. The fitting parameters for (A) are $r_0 = 240\mu m$, $\xi = 80\mu m$, $n_0 = 2.97 \times 10^{11}\mathrm{cm}^{-3}$ and $R = 800\mu m$. The fitting parameters for the left and right panels of (B) are $r_0 = 27\mu m$, $\xi = 23\mu m$, $n_0 = 2.058 \times 10^{12}\mathrm{cm}^{-3}$ $R = 70\mu m$, and $r_0 = 16\mu m$, $\xi = 35\mu m$, $n_0 = 2.645 \times 10^{12}\mathrm{cm}^{-3}$, $R = 73\mu m$, respectively.

In writing down the first line of Eq. (6) we have assumed the temperature is sufficiently low and the trapping potential is harmonic. For the second line of Eq. (6) we have taken into account the universality of the pairing gap near Feshbach resonance[10].

An appealing feature of Eq. (5) is that the microscopic information about the interface (and temperature) enters through a single parameter $\xi(\hat{t})$. In Fig.(2) we plot Eq. (5) and Eq. (6) for typical values of parameters. For $r_0 \gg \xi$, the density difference at the center of the cloud is exponentially small which reveals an equal density core. For small ξ (green line), one can have rather narrow intermediate partially polarized regime (like the Rice group's result). Wider partially polarized regime (like the MIT group's result) can be obtained when ξ is given a larger value (blue line). When ξ increases toward r_0, hence the interface fluctuation becomes more severe, even the center of the core can become visibly partially polarized. Meanwhile the pairing gap Δ_0 remains finite similar to the result of Ref.[4]. Clearly in this theory the partially polarized region is caused by the interface fluctuation rather than a PP normal phase with a vanishing (or smaller) pairing gap.

In Fig.(3) we fit the polarization profile measured by both the MIT and Rice group to the first line of Eq. (5) and Eq. (6). These fits are excellent! Recently a new scheme of tomographic rf spectroscopy invented in MIT[11] gives the promise of measuring the spatial profile of pairing gap $\Delta(r)$, which can be used to test the second line of Eq.(5). For the same sample, if one measures both $\delta n(r)$ and $\Delta(r)$, and provided that both $\delta n(r)$ and $\Delta(r)$ can be well fit by Eq. (5), one can extract the values of r_0 and ξ by fitting these two spatial profiles to these two functions. A falsifiable predication of our phenomenological theory is that the values of r_0 and ξ extracted from these two spatial profiles, $\delta n(r)$ and $\Delta(r)$, shall be the same. This is because in both case it is the same interface fluctuation that gives rise to the smooth spatial profiles.

Now we fill in the details of how to obtain Eq. (5) from the model defined in Eq. (2) and Eq. (3). We will first consider the effects of quantum fluctuation at $T = 0$. Later we shall generalize the result to finite temperature.

Substituting Eq. (4) into Eq. (2) and use the orthonormal relation between different normal modes

$$\int_{\mathcal{S}} \sqrt{g_\phi g_z}\, d\phi dz u_\alpha(z,\phi) u_\beta(z,\phi) = \delta_{\alpha,\beta} \tag{7}$$

we obtain

$$\mathcal{L} = \frac{m}{2} \sum_\alpha{}' \left[\dot{A}_\alpha^2 - \omega_\alpha^2 A_\alpha^2 \right]. \tag{8}$$

Eq. (8) describes a set of independent harmonic oscillators. The corresponding Hamiltonian is given by

$$\mathcal{H} = \sum_\alpha{}' \left[\frac{1}{2m} \Pi_\alpha^2 + \frac{m\omega_\alpha^2}{2} A_\alpha^2 \right], \tag{9}$$

where Π_α and A_β obey the following commutation relation $[A_\beta, \Pi_\alpha] = i\hbar\delta_{\alpha\beta}$. The ground state wavefunction of Eq. (9) is given by

$$\Psi\left[\{A_\alpha\}\right] = \prod_\alpha{}' \left(\frac{m\omega_\alpha}{\hbar\pi}\right)^{1/4} \ \mathrm{Exp}\left[-\frac{\mathrm{m}\omega_\alpha}{2\hbar} \mathrm{A}_\alpha^2\right], \tag{10}$$

where $\prod'$ excludes the breathing mode from the product. Here we explain why some directions ($\hat{t}$ in Eq. (5) and Eq. (6)) are special. For a generic point on the surface specified in Eq. (1), the direction of surface normal does not agree with the vector connecting the center to the point in question. There are special points on the surface, for which these two directions agree. For the surface described by Eq. (1) these special directions are $\pm\hat{z}$ and $\hat{r}$ around the equator at $z = 0$. Let (z_0, ϕ_0) be such a special point,

$$u(z_0, \phi_0) = \sum_\alpha{}' A_\alpha u_\alpha(z_0, \phi_0). \tag{11}$$

Let us first calculate the probability density for the interface to move by amount η

$$P(\eta)=\int\prod_{\alpha}{}'dA_\alpha\delta(\eta-u(z_0,\phi_0))|\Psi[\{A_\alpha\}]|^2 \quad (12)$$

By substituting Eq. (11) for $u(z_0,\phi_0)$, and making use of the identity that

$$\delta(\eta-u(z_0,\phi_0))=\int\frac{d\lambda}{2\pi}e^{-i\lambda[\eta-u(z_0,\phi_0)]}, \quad (13)$$

one can then integrate out each A_α separately and yield

$$\begin{aligned} P(\eta) &= \int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda\eta}e^{-\frac{\lambda^2}{4}\sum_\alpha{}' m\omega_\alpha u_\alpha^2(z_0,\phi_0)/\hbar} \\ &= \frac{1}{\sqrt{\pi\xi^2}}e^{-\eta^2/\xi^2}. \end{aligned} \quad (14)$$

In the above

$$\xi^2=\frac{m}{\hbar}\sum_\alpha{}'\omega_\alpha u_\alpha^2(z_0,\phi_0). \quad (15)$$

For a point r away from the center of the superfluid core, along the vector connecting the center to (z_0,ϕ_0) on the interface, we can calculate the probability $W(r)$ for it to be in the polarized fermi gas region:

$$W(r)=\int_{-\infty}^{r-r_0}P(r')dr'=\frac{1}{2}\left[1+\mathrm{Erf}\left(\frac{r-r_0}{\xi}\right)\right], \quad (16)$$

Therefore the averaged spin polarization is $W(r)n_{\mathrm{p}}(r)$ and the average pairing gap is $(1-W(r))\Delta_{\mathrm{s}}(r)$, i.e. the results of Eq. (5).

Now we consider non-zero temperature. In that case we replace Eq. (12) by

$$P(\eta)=\left\langle\delta(\eta-\sum_\alpha{}'A_\alpha u_\alpha(z_0,\phi_0))\right\rangle, \quad (17)$$

where < ... > denotes the quantum statistical mechanical average. A convenient way to calculate the average of Eq. (17) is to perform Feynman's path integral in imaginary time. Let $A_\alpha(\tau)$ be the Feynman path of each harmonic oscillator in Eq. (9), we have

$$P(\eta)=\frac{\int\prod_\alpha{}'\mathcal{D}A_\alpha(\tau)\ \delta(\eta-\sum_\alpha{}'A_\alpha(0)u_\alpha(z_0,\phi_0))e^{-S}}{\mathcal{Z}} \quad (18)$$

Here $\mathcal{Z}$ is the partition function, and the action S is given by

$$S=\frac{m}{2}\int_0^\beta d\tau\sum_\alpha{}'[\dot{A}_\alpha(\tau)^2+\omega_\alpha^2A_\alpha(\tau)^2]. \quad (19)$$

After using Eq. (13) to represent the delta function, we are left with a quadratic path integral which can be performed exactly to give

$$P(\eta)=\sqrt{\frac{1}{\tilde{\xi}^2\pi}}e^{-\eta^2/\tilde{\xi}^2}, \quad (20)$$

where

$$\tilde{\xi}^2=\frac{2\beta}{m}\sum_\alpha{}'\sum_n\frac{u_\alpha^2(z_0,\alpha_0)}{(\omega_n^2+\omega_\alpha^2)}, \quad (21)$$

and ω_n is the bosonic Matsubara frequency. As $P(\eta)$ has the same form as Eq.(14), $W(r)$ will also have the same form as Eq.(12) which leads to the same function form of Eq. (5).

In conclusion, we have demonstrated that even if there is no true PP phase in bulk, a PP regime can emerge from the quantum and/or thermal fluctuation of the interface. The width of PP regime depends on the strength of the surface fluctuation, and sufficient strong fluctuation can even lead to disappearance of the equal density core, with the pairing gap remaining finite. Though the microscopic description of interface fluctuation remains to be explored, we have shown that, as long as the theory for surface motion is quadratic, there is a universal function form for spin polarization and pairing gap, which can be compared with existing experiments and tested by future experiments as a justification of this scenario.

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